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VORTICITY DISTRIBUTIONS IN UNSTEADY FLOW SEPARATION

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## ABSTRACT

Computational analysis and results are reported for three problems.

(1) Development of a laminar boundary layer on both the windward and leeward sides of a plate which is moved impulsively normal to its plane. The model of inviscid flow outside the boundary layer includes a moving and intensifying line vortex, which approximates the vortex spiral cast off from the edge of the plate.

(2) Mutually-induced movement and interdiffusion of counter-rotating viscous line vortices, simulated by the random-vortex method.

(3) Development of flow separation on a slender elliptical cylinder, which is impulsively set into rotation around its central axis, also simulated by the random-vortex method.

## VORTICITY DISTRIBUTIONS IN UNSTEADY SEPARATING FLOWS

### 1. Introduction

On 15 May 1986, Grant No. AFOSR-86-0168 was issued to the University of California at Berkeley, in response to a proposal for the "Development of Systematic Criteria for the Prediction of Enhancement or Loss of Lift in Unsteady Aerodynamics". Funds were established at the University and work started, at the beginning of August. A graduate research assistant, Michael Dooley, was employed and assigned some warm-up tasks which would teach him the Random-Vortex Method.

The initial guiding idea of the proposal was that one could hope to foresee the conditions under which large concentrations of vorticity, formed as the result of a sudden pitching or heaving motion of a wing, would hold together and stay close to the wing, or would be shredded apart and carried off downstream, without doing an experimental test or detailed numerical simulation for each case of interest. This idea is based on theoretical demonstrations of the importance of the ratio of principal rate of deformation to vorticity in determining the outcome, and on the hope that one could acquire a feeling for the factors which determine the size of this ratio, by studying a few well-selected sample calculations. It was proposed to do these calculations with the Random-Vortex Method, a scheme which automatically produces a vivid picture of the evolving vorticity distribution, and which has a good record for realistic simulations of separation on curved surfaces.

As a result of a long consultation with Dr. Larry Carr in the Spring of 1987, we decided to pay somewhat greater attention to the early development of separation, for which boundary-layer analysis is relevant. I undertook some sample calculations of unsteady boundary layers, while Mr. Dooley continued with Random-Vortex calculations.

Because the numerical work for boundary-layer calculations is comparatively easy and cheap, we already have a significant result, which was presented at the Unsteady Separated Flow Workshop at the Air Force Academy last July, and which is ready for publication. This is described in Section 1.

Mr. Dooley's two warm-up problems have produced some intrinsically interesting results, and have increased our familiarity with the Random Vortex Method. These are described in Sections 3 and 4. The relevance of this initial work to the objectives of the proposal, and our plans for future work, are sketched in Section 5. An attempt is made there to relate our work to the other activities reported at the Workshop.

## 2. Impulsively-started laminar flow around a sharp edge

A preliminary report of this study was given at the workshop; a substantially complete account was sent in for publication in the workshop proceedings. The version enclosed with this report has a few more references and a slightly refined discussion, resulting from consultations and correspondence with Professors N. Rott at Stanford, H.K. Cheng at U.S.C., J.C. Williams III at Auburn, J.D.A. Walker at Lehigh, and Dr. Dale Pullin of Queensland, Australia.

In the context of our proposal, this study was part of an effort to see whether there may be some simple way to initialize a computational flow field at a time just after a sudden change of airfoil incidence, knowing only the state of the boundary layer (presumably attached) before the change, and the displacement of the potential flow along the wall during the change. Such a procedure was suggested many years ago by W.R. Sears, and is mentioned in the book on unsteady viscous flow by D.P. Telionis.

I was soon convinced, especially by the calculation of H.J. Lugt and S. Ohring (JFM 79 127-156 (1977)) that a typical potential flow history would involve the casting-off of a starting vortex from the trailing edge, and that this vortex would grow to significant strength and move significantly away from the edge by the time a certain angular displacement was reached, no matter how rapid the angular displacement. The effect of such a vortex on the boundary layer had, as far as I know, never been studied. My model problem was an attempt to study it, which soon acquired a fascination of its own. I have not yet made much of it, in the context that inspired it.

### 3) Mutually-propelling, interdiffusing line vortices

To familiarize Mr. Dooley with the Random-Vortex Method in the simplest possible context, we have undertaken to simulate the evolution of a viscous flow which consists initially of two counter-rotating, infinitely concentrated parallel line vortices in an infinite surrounding fluid. The initial vortices are equally strong, having circulation  $\pm \Gamma$ , they are initially separated by a distance  $H$ . From these parameters, we derive natural scales for velocity,  $\Gamma/H$ , and for time,  $H^2/\nu$ . If density and viscosity are constants, the flow will be governed by the value of a single dimensionless parameter,  $\Gamma/\nu$  where  $\nu$  is the kinematic viscosity.

If the flow is hydrodynamically stable, the vortices will diffuse, while drifting along in a direction normal to the line connecting their original positions. The drift velocity will gradually fall away to zero, as the vortices weaken each other by interdiffusion. This problem has been recently attacked by Professors Rott and Cantwell at Stanford, by analysis of approximations appropriate to either very small, or very large, dimensionless times.

In the Random-Vortex Method, the circulation of each initial vortex is apportioned equally among a fairly large number,  $N$ , of computational elements called vortex blobs. The vorticity assigned to each element is spread out in an axially-symmetric distribution, designed to avoid the singularity associated with a potential line vortex. For example, the vorticity may have uniform strength within a circle of radius  $\delta$  and be zero elsewhere. As the simulation evolves through a small discrete time step,  $\Delta t$ , each blob is given

- 1) a convective displacement, due to the velocity induced at its axis by all the other blobs, and
- 2) a diffusive displacement, drawn at random from a Gaussian distribution with zero mean and variance  $2\nu\Delta t$ .

The simulated flow field thus depends on the values of three computational parameters,  $N$ ,  $\delta/H$ , and  $\nu\Delta t/H^2$ , besides that of the physical parameter  $\Gamma/\nu$ . Because of the stochastic simulation of viscous diffusion, the outcome also depends on the point of entry to the computer's string of random numbers. Finally, the simulated results depend on the choice of time-integrating scheme, and on techniques which can be employed to reduce the number of operations required for accurate evaluation of the velocity induced on each element.

Although much theoretical and computational study has been devoted to issues of convergence of vortex-blob simulations of non-diffusive flows, one cannot yet turn to the literature to discover what values of our computational parameters would guarantee a certain level of accuracy in simulation of a flow which involves, as this does, a strong interaction of convection and diffusion. Neither can one be sure of the effects of various choices of integration scheme or of schemes to economize on calculations. Mr. Dooley is exploring these issues.



Actually, this warm-up problem has considerable intrinsic, if academic interest. It may also be not so far afield from the main theme of our research as might appear at first glance. The principal question of interest is whether the two vortices merely blend together into a sort of decaying viscous vortex doublet, which drifts as a decelerating, but compact unit; or whether, perhaps for higher values of  $D/v$ , part of each vortex is expelled at some time, to trail along behind the main vortex pair. If the latter scenario appears, we should like to know whether we could have predicted it, at least somewhat in advance, by observing locally increasing ratios of deformation rate to vorticity. Also, nothing at all is known about the stability of this flow, even against entirely two-dimensional disturbances. The traditional framework of instability analysis seems to offer little hope of a result, but the Random-Vortex Method may offer an interesting alternative approach.

If it is, as we hope, possible to show the existence of a range of computational parameters within which further refinements of parameters (increase of  $N$ , decrease of  $\delta/H$  and or  $\nu \Delta t/H^2$ ) and/or changes of the point of entry to a random-number string produce no significant changes in the trajectory of the centroid of each collection of blobs, we should conclude that the flow is physically stable, at least against two-dimensional disturbances. We assume that, since the Random-Vortex Method continually injects small accidental disturbances of the vorticity pattern, which will be different for each choice of computational parameters and for each sequence of random numbers, no demonstration of computational convergence will be possible for a physically unstable flow.

What we have discovered so far was somewhat unexpected, but not too hard to explain. At the very early times, when the vortex blobs from each initial point form a compact cloud, the motion of the centroid of each cloud is significantly

affected by statistical fluctuations of the mean of the random displacements drawn at each time step. Although these are drawn from a population with zero mean, the experimental mean values, for samples of size  $N$ , from a population with variance  $2 \Delta t/N$ .

At these same early times, the convective displacement of the centroid of a cloud equals about  $\Gamma \Delta t / 2\pi H$ . The ratio of a likely statistical displacement of the centroid,  $\Delta X_s$ , to this convective displacement,  $\Delta X_c$ , is thus

$$\frac{\Delta X_s}{\Delta X_c} \approx \left(\frac{\nu}{\Gamma}\right) \sqrt{\frac{1}{N \Delta \tau}},$$

where  $\Delta \tau = \nu \Delta t / H^2$  is the dimensionless time step. Thus, at least at early times, the random-walk simulation of viscous diffusion will significantly distort the trajectories of the centroids, especially for small values of  $\Gamma \nu$ , unless  $N \Delta \tau$  is kept suitably large.

The question that arises immediately is whether these distortions are significantly amplified by convection, i.e. is the flow hydrodynamically unstable? Our tentative conclusion is that they are not amplified if  $\Gamma \nu = 10$ , they may be slightly amplified if  $\Gamma \nu = 100$ , and they are dramatically amplified if  $\Gamma \nu = 1000$ . These conclusions came from a small collection of calculations, in all of which  $N \Delta \tau$  equalled either 1 or 1/4. It is interesting that the symptoms of amplification, mainly a growing lack of symmetry about the  $x$  - axis, were strongest when  $\Delta X_s / \Delta X_c$  was least.

We have taken one obvious step to investigate this phenomenon, by forcing the mean random displacement of each vortex cloud to be zero at each time step. This was done by subtracting the observed, non-zero, mean displacement from the displacement of each blob. To our surprise, this had almost no effect on the evolution of the flow field, including the gross asymmetries observed when

$U/v = 1000$ . The next step will be to enforce symmetry about the x-axis, by letting every blob below the axis move as a mirror image of a blob above it.

At the present time, these computations are in abeyance while the computer programs are restructured to take better advantage of the parallel-processing capability of the CRAY computer. By consultation with Dr. Scott Baden of Lawrence Berkeley Laboratory, we have ascertained that we may expect to speed the calculations up by a factor as much as ten.

The qualitative behavior of the vortex clouds seems the same at  $U/v = 10$  and 1000, with the vorticity moving as a compact cloud. In all runs done for  $U/v = 1000$ , a distinct tail of vorticity is left behind the main cloud.

Figure 1 shows a typical, nearly symmetric distribution of vortex blobs when  $U/v = 100$ , and  $vt/H^2 = 0.5$ . Figure 2 shows the x-progress of the centroid of vorticity for five different runs at  $U/v = 100$ , and one run at  $U/v = 10$ . Two runs for  $U/v = 100$  used  $N = 1000$ ; the other three used  $N = 250$ . The run at  $U/v = 10$  used  $N = 250$ . All used  $\Delta\tau = 1/1000$  and  $\delta/H = 1/10$ . Different results for the same values of  $U/v$  and  $N$  result from different points of entry to the computer's random-number string. Figure 3 shows the temporal evolution of the blob pattern for  $U/v = 1000$ ,  $N = 1000$ ,  $\Delta\tau = 1/1000$ .

Because of the intrinsic interest of this problem, both for an understanding of the random-vortex method, and as a model of an unfamiliar kind of hydrodynamic instability, it may be developed into a Master's thesis for Mr. Dooley. He will, however, continue toward the main objective of an investigation of unsteady separating flow.

A second warm-up problem, substantially closer to the real thing because it involves the imposition of no-slip and no-penetration conditions on a non-trivially moving solid wall, has been worked on in parallel with the vortex-pair

problem. It was selected for its representative degree of difficulty, and because it has been well analyzed previously by finite-difference methods.\*

We have chosen to meet the no-penetration condition by the use of image vortex blobs in a conformally-transformed flow, in which the wall is a circle. The alternative, of using singularity panels inside the ellipse in the physical plane, is in many ways attractive, but was rejected because of our fear that it would not work well for airfoils with sharp trailing edges.

Many new computational-design issues are raised by the need to meet the no-slip condition on a surface with widely variable curvature. This condition is satisfied by each time step, at a discrete number of test points on the surface, by the introduction of sheet-like vorticity elements. These elements, like the blobs, can be designed in various ways. They are used only in a thin layer next to the surface, affect the convection of only those other sheets that lie between them and the wall, and diffuse by random walk only in the direction normal to the wall. When a sheet moves far enough from the wall, so that the probability of its moving back to it by a single random step is small, it undergoes a metamorphosis into a blob. We have been distributing the zero-slip test points uniformly around the circle in the transformed plane, because that brings them close together near the separation points in the physical plane.

After a series of debugging runs, our application of the Random Vortex Method to this problem seems to be successful. Figure 4 shows the disposition of vortex blobs and sheets when the ellipse has rotated through an angle of three degrees. The points at which the no-slip condition is enforced are indicated by filled circles on the surface of the ellipse. At the time of the pic-

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\*Lugt and Ohring (1973) op. cit.

ture, there are 1550 vortex elements. This number increases steadily with time - the initial number of computational elements (all sheets) was 324. The number of time steps used to reach this state was 10, requiring 228 CRAY seconds.

#### 4) Summary, and prospects for the original proposal.

I must admit that I think it unlikely that our line of investigation is going to make an early contribution to the practical objectives of the program that was reviewed at the 1987 Workshop on Unsteady Separated Flow. It is taking us too long to attain a trustworthy computational capability, even for flows which others have already simulated by other means.

Our idea, that we might be able to foresee the future evolution of a vorticity pattern without detailed calculation, by watching the evolution of the ratio of strain rate to vorticity, has not yet proved to be very helpful. It did suggest, correctly, that the interdiffusing vortex pair of the first warmup problem would develop a trailing wake of vorticity at some value of  $\Gamma/\gamma$ , but it did not forewarn us of the dramatic instability of the trajectory of the vortex pair.

On the other hand, the demonstration of the trajectory instability seems to be of considerable intrinsic interest. It may be related to the observed meandering of jets and plumes or to the onset of asymmetric vortex shedding behind a suddenly accelerated, nominally symmetric aerodynamic body.

I believe that our work on boundary layers near an edge that is casting off a vortex provides a nice supplement to the work of Professor Cebeci\*, who has been focusing on transient boundary layers near a rounded leading edge. Two-dimensional, transient, laminar boundary layers can be faithfully described computationally, up to the time of separation.

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\*Reference to his talk at the Colorado Springs Workshop.

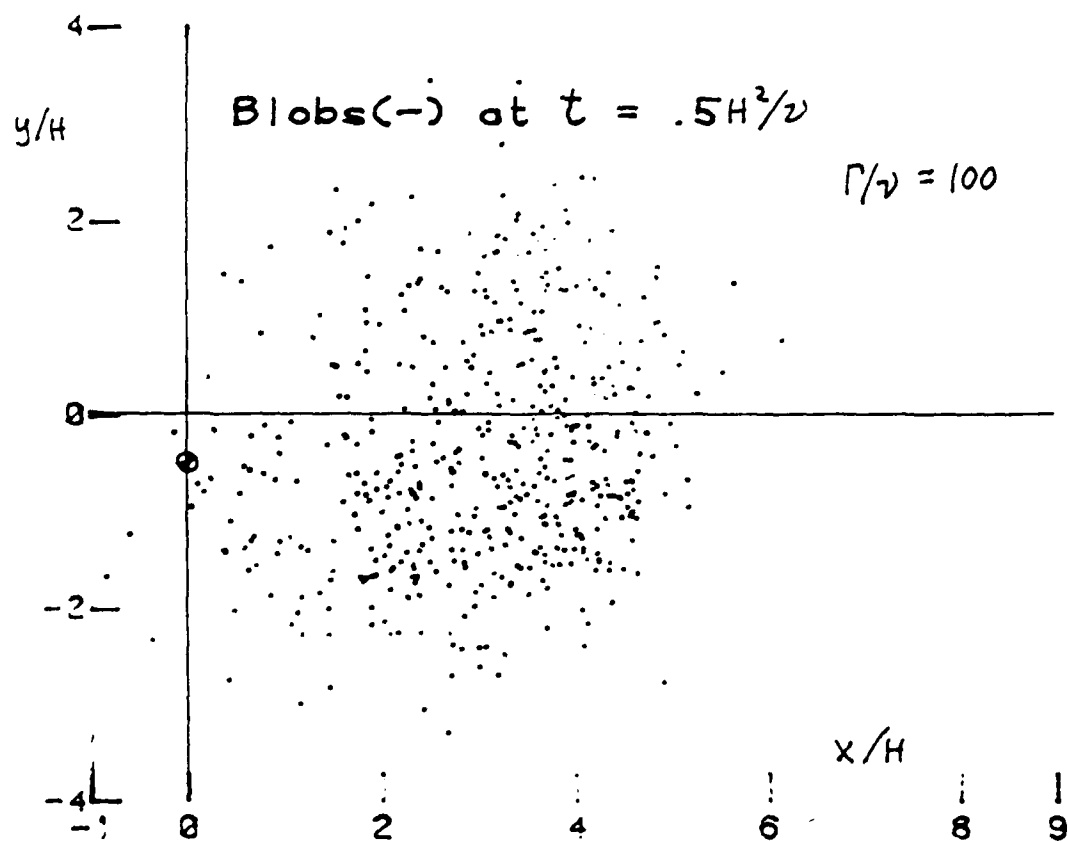
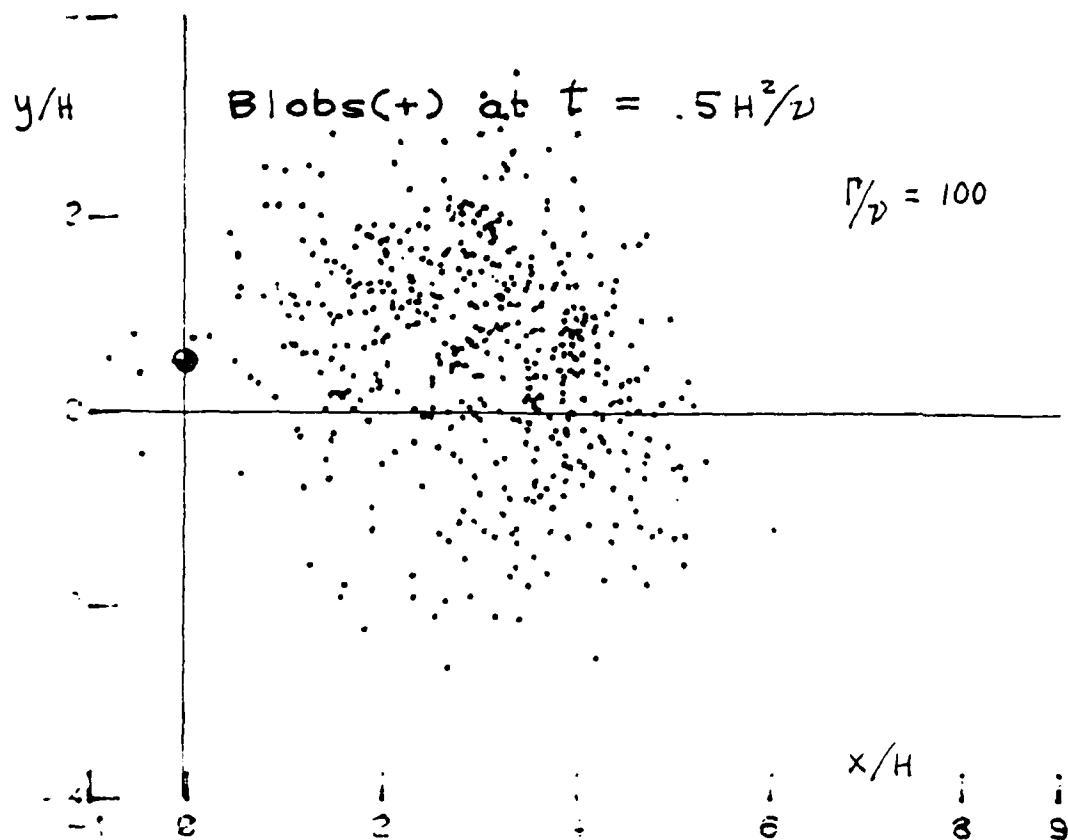


FIGURE 1

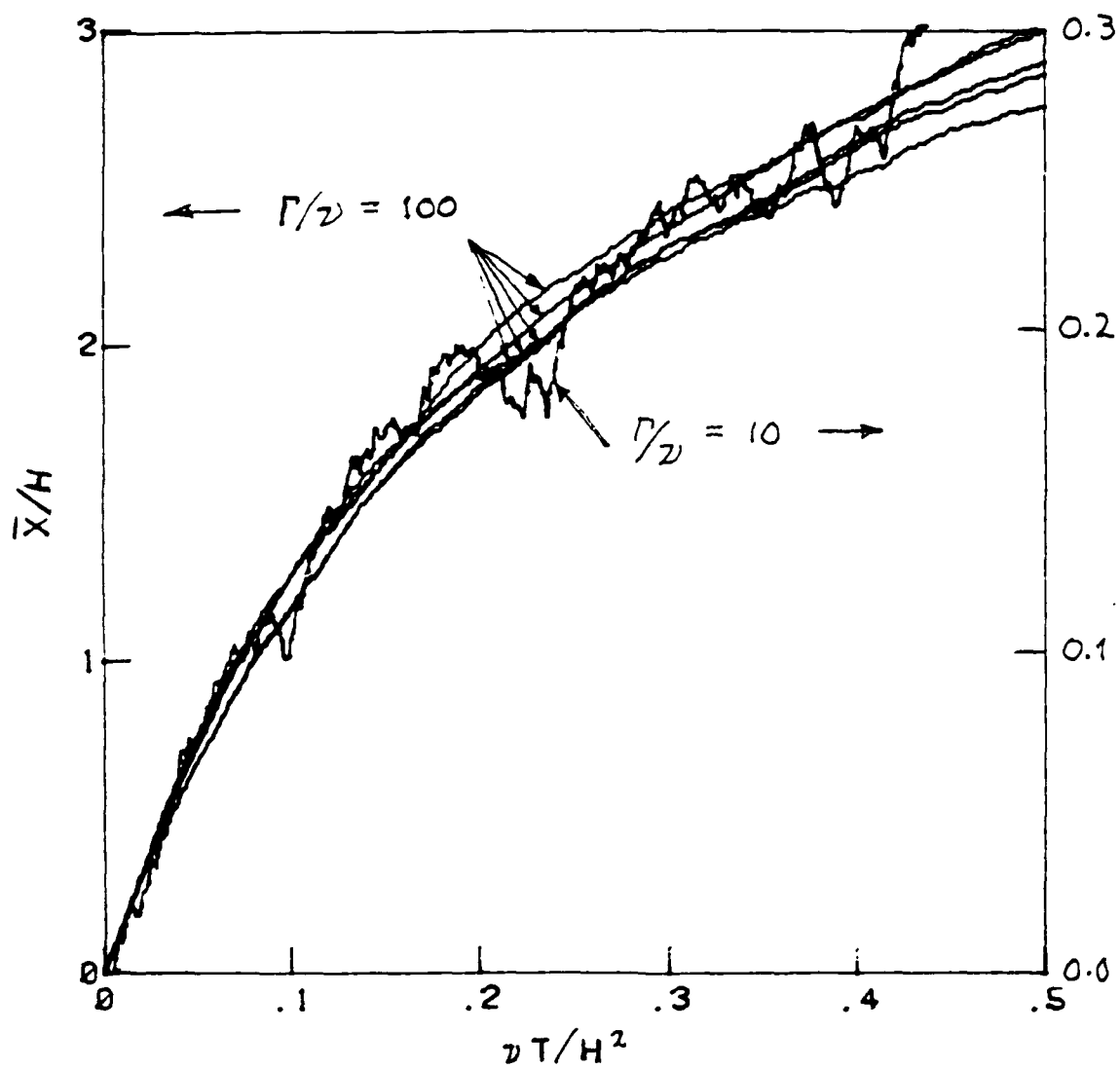


Figure 2

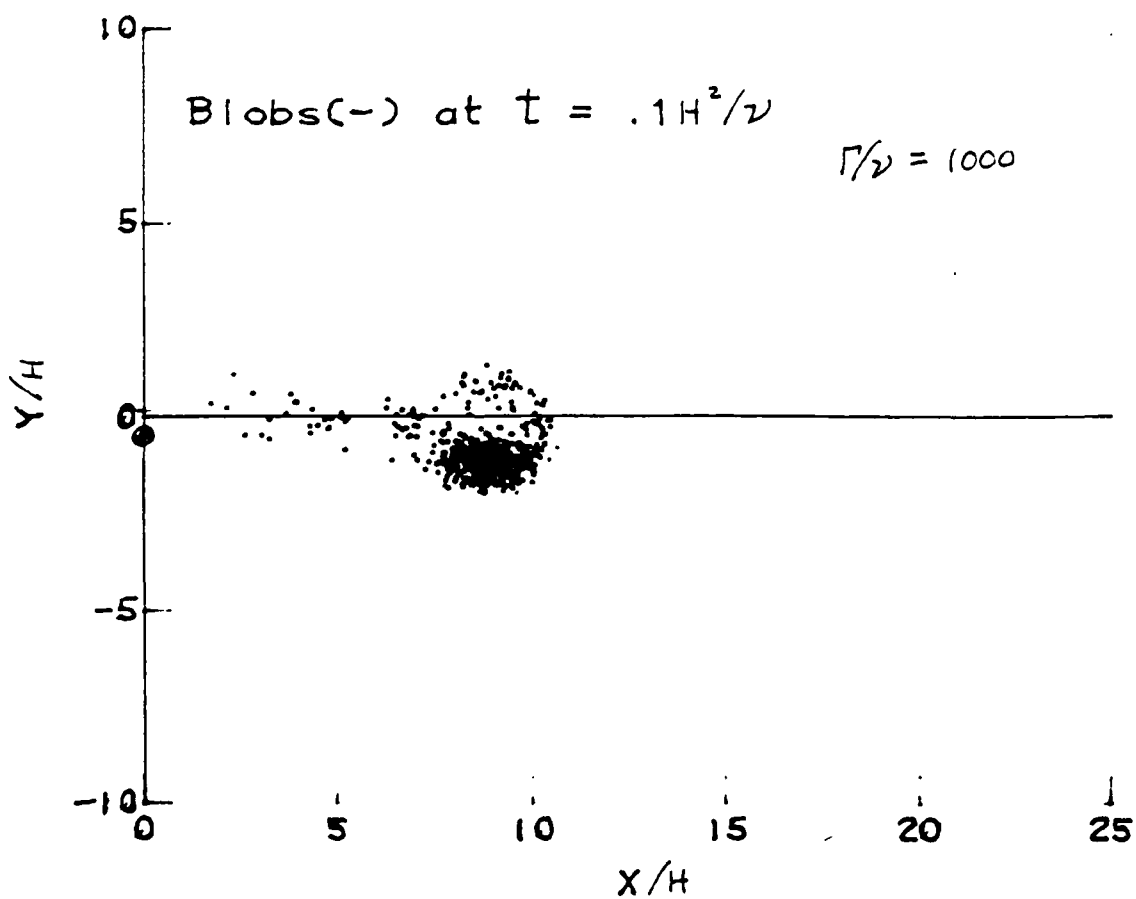
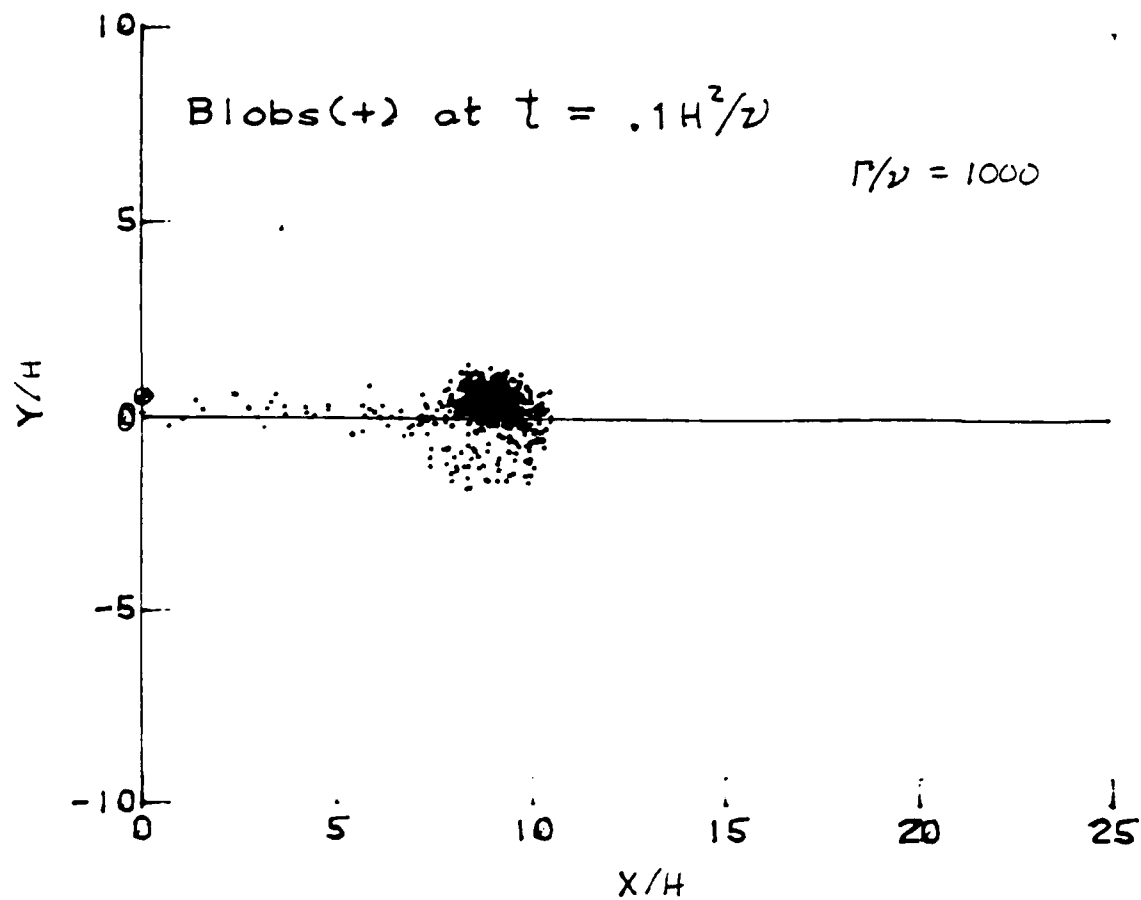


FIGURE 3 (a)



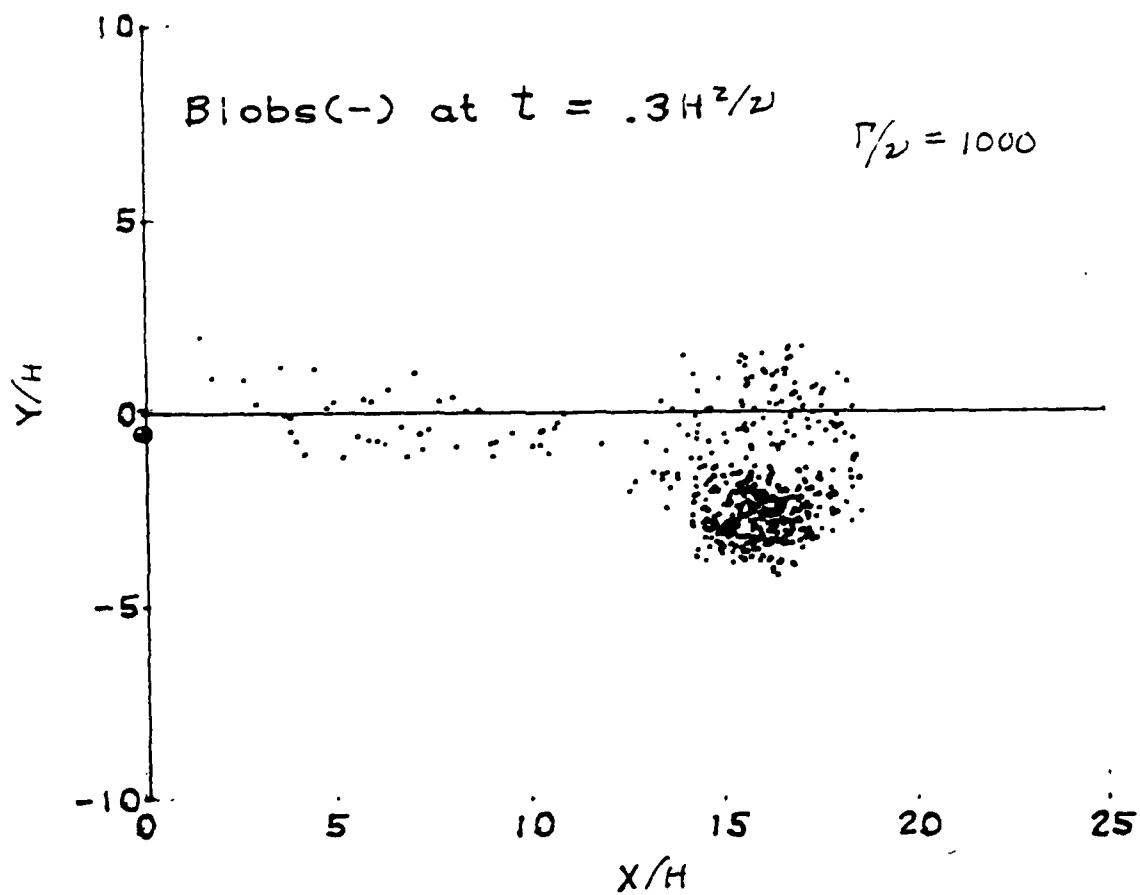
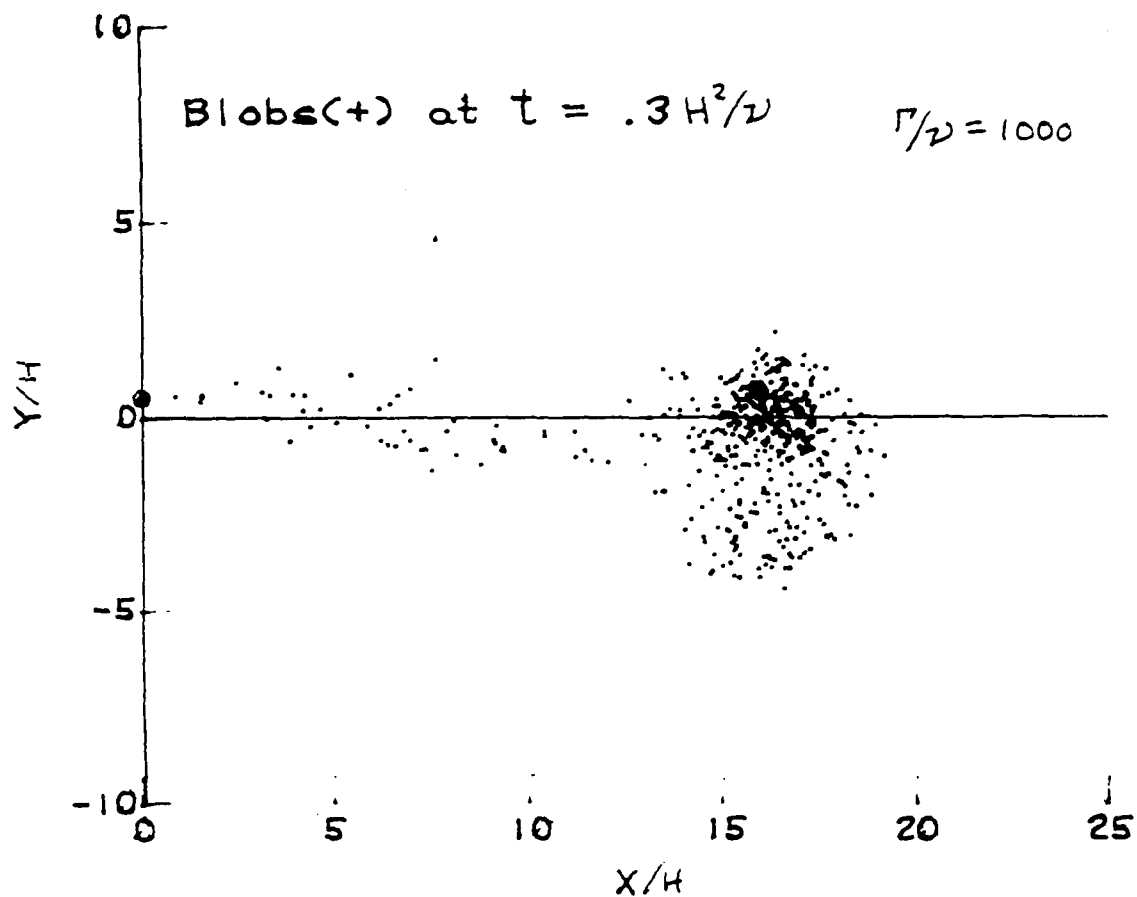


FIGURE 3 (b)

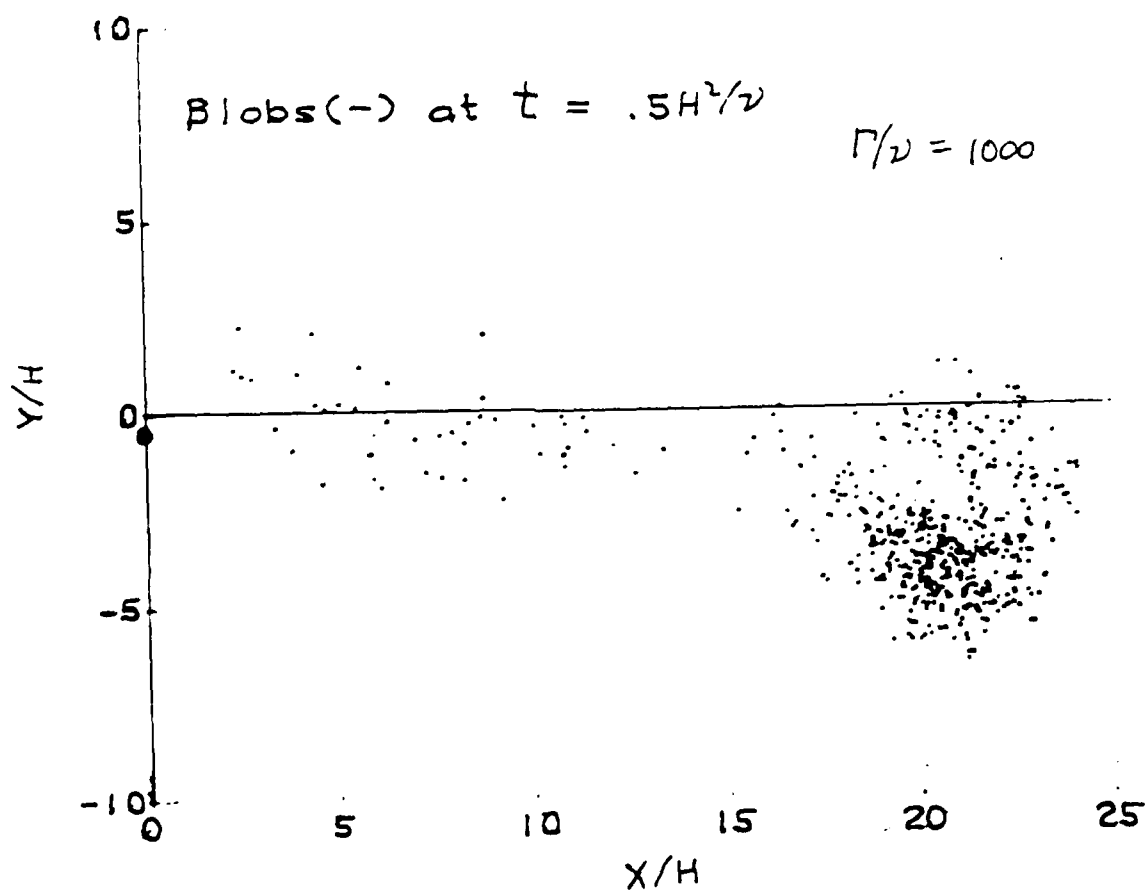
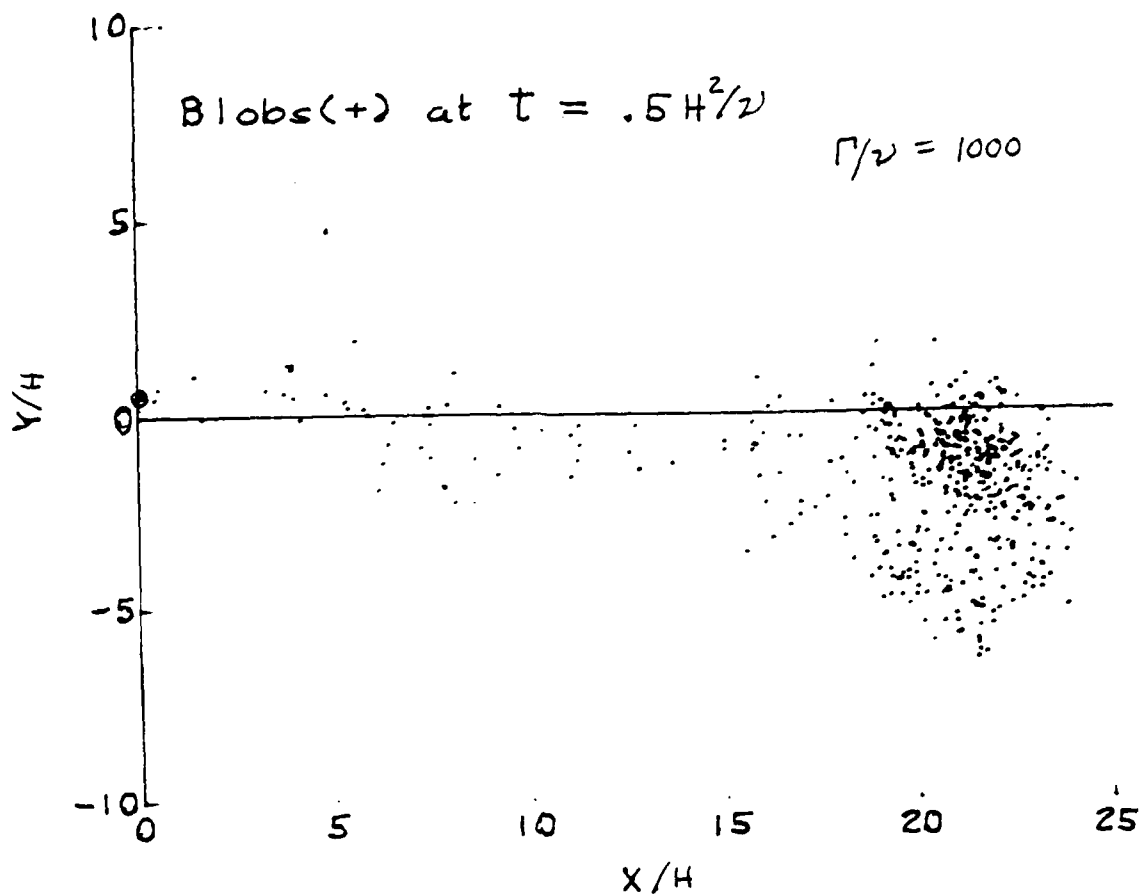
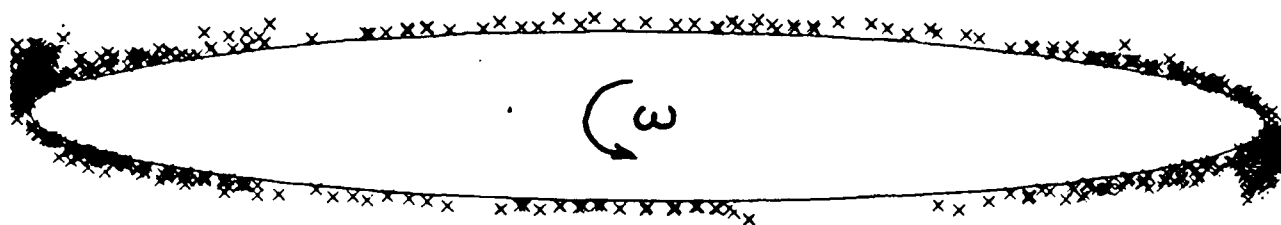


FIGURE 3(c)



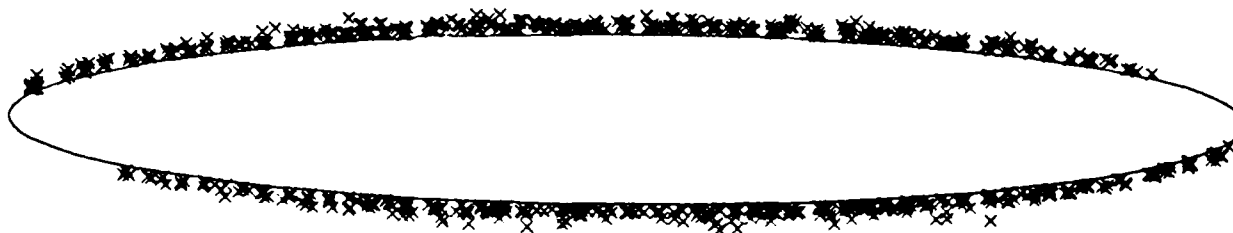
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0.00000

0.71783

NEGATIVE BLOBS

1



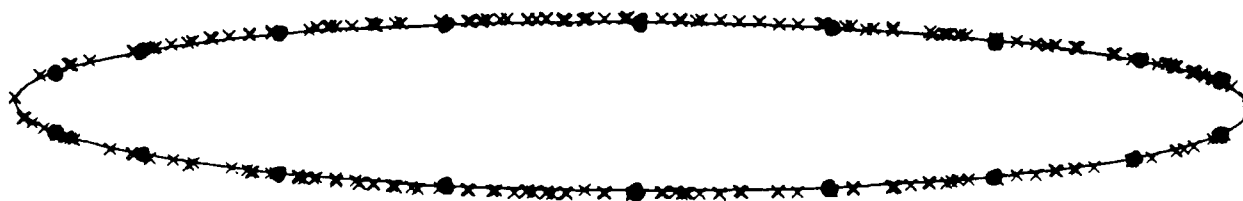
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0.00000

0.71783

POSITIVE BLOBS

1



● Point at which no-slip condition is enforced

-0.71783

0.00000

0.71783

SHEETS

Figure 4

## APPENDIX

### Vectorization of Code for Random-Vortex Method

Introduction: When the Random-Vortex code that we had been using on an IBM 3091 was installed on the local CRAY XMP, initial results were disappointing. Specifically, run times were longer than expected.

After consultation with Dr. Scott Baden, who has great experience with the execution of discrete-vortex algorithms on parallel processors, and further consultation with the local CRAY staff, we were able to rearrange the code so that it could be more comprehensively vectorized, with a resulting five-to ten-fold increase in speed.

Our implementation of the Random Vortex Method (RVM) includes, like most modern RVM codes, a strategy for grouping vortex elements that occupy a given spatial cell, so that their effect on elements in distant cells may be efficiently computed. This has well-known advantages, but the necessary sorting of elements, to decide which cell they are in and which other cells are close or distant neighbors, involves the use of many IF statements which inhibit vectorization. Also, at least on our local CRAY, a DO-loop in which there is more than one cell for a random number cannot be vectorized.

When these difficulties are recognized, the code can be rearranged to avoid as many of them as possible. For example, we calculate induced velocities for all vortex elements in a DO-loop which cannot be vectorized, and then calculate the resulting convective displacements in a second DO-loop, which can be vectorized.

We believe that our present code is vectorized to a nearly optimal degree, with one conspicuous exception. Our choice of a "core function" for each vortex element, which makes the mutual interaction of such elements non-singular as their separation approaches zero, involves a discontinuous distribution of vorticity:  $\Omega = \Gamma/\pi\sigma^2$  if  $r \leq \sigma$ ,  $\Omega = 0$  if  $r > \sigma$ . This choice is implemented in the code by another IF statement sending the calculation on to different formulas depending on the magnitude of the separation.

This last IF statement can be avoided by use of one of several continuous distributions of vorticity for an element. Several examples, by Krasny, and by Beale and Majda, are in the literature. They require slightly more calculation per element, but allow a greater degree of vectorization.